NASA TECHNICAL NOTE

NASA TN D-4997

NASA TN D-4997

ERROR ANALYSIS OF INPUT-NOISE-TEMPERATURE MEASUREMENTS OF LOW-NOISE AMPLIFIERS

by Hans-Juergen C. Blume

Langley Research Center
Langley Station, Hampton, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JANUARY 1969

ERROR ANALYSIS OF INPUT-NOISE-TEMPERATURE MEASUREMENTS OF LOW-NOISE AMPLIFIERS

By Hans-Juergen C. Blume

Langley Research Center
Langley Station, Hampton, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 – CFSTI price $3.00

ERROR ANALYSIS OF INPUT-NOISE-TEMPERATURE MEASUREMENTS OF LOW-NOISE AMPLIFIERS

By Hans-Juergen C. Blume
Langley Research Center

SUMMARY

The determination of input noise temperatures of low-noise amplifiers, such as masers, parametric amplifiers, and tunnel-device amplifiers, should be independent of the characteristics of the measuring system. The method presented provides means of determining all effects which significantly influence measurement of the output-noise-power ratio (Y-factor) of a typical low-noise-amplifier test setup. The error analysis of the Y-factor method illustrates the error propagation which occurs in the derivations of the input noise temperature and gives an estimate of the maximum error. The general concepts used in determining these influences and the error propagation are applied to reflection-type amplifiers. Numerical examples demonstrate the value in considering all elements which (a) affect the calibrated noise power along the transfer path from the noise source to the amplifier under test and (b) add noise power along the path from the amplifier to the noise indicator. The error analysis applied to the examples revealed that for minimization of the input-noise-temperature error, accurate knowledge of the losses and temperatures is more important than obtaining extremely high ratios of the hot- to cold-noise-source temperatures. A comparison between the input noise temperature determined by considering all influencing effects and the input noise temperature determined without considering these effects for a typical low-noise measuring system for reflection-type amplifiers revealed a considerable difference.

INTRODUCTION

The input noise temperature of a device is frequently determined by measuring the output-noise-power ratio Y with two calibrated noise sources applied at the input. This procedure is called the Y-factor method. The output noise levels depend not only upon the input noise powers and the internally generated noise of the device under test but also upon the power-transfer characteristics of the network elements before and after the device under test. Normally, the noise figure and gain of the amplifier stage following the device under test are considered and all other effects are neglected. These effects do not introduce significant errors as long as the input noise temperatures are higher than 1000° K. However, the development of low-noise, reflection-type amplifiers, such as masers, parametric amplifiers, and tunnel-device amplifiers, has reached a state of such low excess noise levels that the determination of the input noise temperature becomes very inaccurate if the influencing effects of the network elements before and after the amplifier under test are not taken into account.

It is the purpose of this paper to point out possible effects which influence the accuracy of the Y-factor method and to include these effects in the determination of the input noise temperature. This paper also evaluates the accuracy of the input noise temperature determined by this method with an error-propagation calculation which takes into account the measuring inaccuracies of these effects. This evaluation method is applied to a typical Y-factor measuring setup for negative-resistance amplifiers. Numerical examples for a test setup used to measure the input noise temperature of liquid-nitrogen-cooled parametric amplifiers demonstrate the necessity of considering all effects which influence the determination of the input noise temperature from the measured Y-factor and indicate that errors as large as 100 percent may exist.

The author wishes to thank Prof. Dr.-Ing. Habil. Kurt Lamberts, Associate Professor of Technische Hochschule, Braunschweig, West Germany, for valuable discussions about the test set.

SYMBOLS

a	relative error of mismatch factor α
b	phase angle, degrees
F	noise figure
G	gain or gain factor
g	relative error of gain factor G
$\frac{1}{g_{C1}}$	isolation from port 3 to port 2 of circulator 1
k	Boltzmann's constant
L	dissipation loss
N	noise power, watts

N_{C2}	noise power from matched load of circulator 2, watts
N'_{C2}	portion of N_{C2} reflected toward amplifier under test, watts
N''_{C2}	leakage of noise power N_{C2} from port 3 to port 2 of circulator 1, watts
N'	available output noise power of dissipative elements, watts
N''	available output noise power of reflective elements, watts
P	measured output noise power, watts
P_C	measured output noise power when cold noise source is connected, watts
P_H	measured output noise power when hot noise source is connected, watts
p	error in percent
T	noise temperature, degrees Kelvin
T_{C2}	equivalent noise temperature caused by matched load at circulator 2, degrees Kelvin
T_e	effective noise temperature, degrees Kelvin
T_f	false noise temperature, degrees Kelvin
t	ambient temperature, degrees Kelvin
t_{C2}	ambient temperature of matched load at circulator 2, degrees Kelvin
t_o	room temperature, degrees Kelvin
x,y,z	variables
Y	measured output power ratio of two different noise sources
α	mismatch factor, except in equation (3) where α is damping constant

β	relative error of phase angle b
Γ	reflection coefficient
γ	relative error of reflection coefficient Γ
Δ	absolute error, except for bandwidth Δf
Δf	bandwidth, hertz
ϵ	relative error of Y-factor
ϑ	relative error of ambient temperature t
λ	relative error of dissipation loss L
ξ	relative error, general
τ	relative error of noise temperature T

Subscripts:

C	quantity related to cold noise source and line through to switch
c	modified quantity related to cold noise source and line through to switch
g,k,l,m	reference planes
H	quantity related to hot noise source and line through to switch
h	modified quantity related to hot noise source and line through to switch
N	noise sources, general
n	reference-plane index; n = 1,2,3,. . .
x	modified quantity related to line between switch and amplifier under test
z	modified quantity related to line between amplifier under test and indicator

Superscripts:

$j = \sqrt{-1}$

l length of line, meters (see eq. (3))

Y-FACTOR METHOD

The Y-factor method, as described in reference 1, pages 34 to 36, and reference 2, is the most common method for the determination of the input noise temperature of low-noise amplifiers because of the well-defined noise-source temperatures. The accuracy of determining the input noise temperature of low-noise amplifiers is closely related to the measuring setup, which is, in the general case, a chain of cascaded networks. The output noise power P is determined by the gain and equivalent-noise-temperature parameters of the chain shown in figure 1 as follows:

$$P = k\,\Delta f\Big[T_N\left(G_1G_2 \ldots G_k \ldots G_m\right) + T_1\left(G_2G_3 \ldots G_k \ldots G_m\right) + T_2\left(G_3 \ldots G_k \ldots G_m\right) + \ldots + T_k\left(G_{k+1} \ldots G_m\right) + \ldots + T_m\Big] \tag{1}$$

where k is Boltzmann's constant, Δf represents the bandwidth of the cascaded networks, T_N is the noise temperature of the noise source at the beginning of the chain, and T_1 to T_m are equivalent output temperatures of the noise power which is generated internally in each two-port network. The gain factors G_1 to G_m may be applied to active or passive networks with reflection losses, dissipation losses, or gain. When the gain-factor products are replaced by the multiplication series notation, equation (1) changes to

$$P = k\,\Delta f\left(T_N \prod_{n=1}^{m} G_n + T_1 \prod_{n=2}^{m} G_n + \ldots + T_k \prod_{n=k+1}^{m} G_n + \ldots T_m\right) \tag{2}$$

In the evaluation of G_n and T_n for passive networks, it is of special importance to differentiate between reflection losses and dissipative or ohmic losses. Ohmic losses, produced by lossy elements such as lines, not only decrease the transmitted noise power but also generate their own noise power, in accordance with the relation of reference 3, page 16

$$N' = Ne^{-\alpha l} + \left(1 - e^{-\alpha l}\right)k\,\Delta f\,t_n \tag{3}$$

where N' denotes the available output noise power; N, the available input noise power; t_n, the ambient temperature of the lossy line element; and $e^{\alpha l} = L$, the dissipation power loss as defined in reference 4, pages 462 to 466. The symbol α is the generally used symbol for damping constant and is not to be mistaken for the mismatch factor in the equations that follow. The first term of the right-hand side of equation (3) represents the attenuation due to ohmic losses and the second term represents the noise power which is generated internally. The reflection losses, however, only decrease the transferred noise power by mismatching and form no additional noise source. The available output noise power after reflection can be expressed according to the relation

$$N'' = N\alpha \tag{4}$$

where α denotes the mismatch factor (ref. 5, p. 191). Multiple reflections between reflection points are neglected because this influence is very small if the mismatch factor is kept close to unity, as is usually done for low-noise-temperature measuring setups.

Since the reflection losses affect the propagating noise power differently than do the dissipation losses, it is necessary to divide the measuring setup into cascaded two-port networks, as indicated in figure 1. The number of two-port networks depends on the number of reflection locations and on the number of sections with different gain in the measuring setup. Possible reflection locations are, for example, waveguide switches, connectors, waveguide junctions, input and output ports of circulators, and attenuators. The measuring setup is divided as follows: The lossy section between any two consecutive reflection locations is an independent two-port network where the first reflection is included in this network and the second reflection is included in the following network. Figure 2 shows the division of the first part of the chain, with the noise source and following waveguide sections. The noise source with noise temperature T_N has the available noise power

$$\dot{N} = k\,\Delta f\,T_N \tag{5}$$

in reference plane 1. The following network 1 has the loss factor L_1, the mismatch factor α_1, and the ambient temperature t_1. The noise power delivered to reference plane 2 is, according to equations (3) to (5),

$$N_2 = k\,\Delta f\,T_N \frac{\alpha_1}{L_1} + \left(1 - \frac{1}{L_1}\right) k\,\Delta f\,t_1 \tag{6}$$

By substituting

$$\frac{\alpha_1}{L_1} = G_1 \tag{7}$$

and

$$t_1\left(1 - \frac{1}{L_1}\right) = T_1 \tag{8}$$

equation (6) reduces to

$$N_2 = k\,\Delta f\left(T_N\,G_1 + T_1\right) \tag{9}$$

The determination of the noise power after m two-port networks leads to equation (2). If the kth network is an amplifier with noise figure and gain, equation (7) changes to

$$G_k = G_k'\,\alpha_k \tag{10}$$

where G_k' is the amplifier gain under matched conditions, and equation (8) changes to

$$T_k = \left(F_k - 1\right) t_o\,G_k \tag{11}$$

where F_k is the noise figure of the kth network, and t_o is the room temperature. For reflection-type amplifiers, the mismatch factor α_k is already included in the gain.

The mismatch factor α can be determined from the measured reflection coefficients. In general, it is possible to open the line at the reflection location and measure the reflection coefficients in both directions, as illustrated in figure 3. The short terminations are provided to measure the loss factors L_{l-1} and L_l and the reflection coefficients Γ_{l1} and Γ_{l2} of the lth network with the method described in reference 4, pages 471 to 476. The numerical indices of the reflection coefficients of the lth network apply as indicated in figure 3. The mismatch factor is, according to reference 6, page 866,

$$\alpha_l = \frac{\left(1 - \left|\Gamma_{l1}\right|^2\right)\left(1 - \left|\Gamma_{l2}\right|^2\right)}{\left|1 - \Gamma_{l1}\,\Gamma_{l2}\right|^2} \tag{12}$$

All terms in equation (2) are reduced to quantities which can be measured and/or calculated.

As previously indicated, the Y-factor represents the ratio of the measured output noise powers of two different noise sources; that is,

$$Y = \frac{P_H}{P_C} \tag{13}$$

where P_H and P_C are the indicated noise powers at the end of the chain of networks, as shown in figure 4. The two calibrated noise sources with noise temperatures T_C and T_H have the available noise powers

$$\left.\begin{aligned} N_C &= k\,\Delta f\,T_C \\ N_H &= k\,\Delta f\,T_H \end{aligned}\right\} \tag{14}$$

These noise powers are changed according to the network parameters G_{1C} and T_{1C}, G_{2C} and T_{2C}, to G_{gC} and T_{gC} in the setup from the cold noise source to the waveguide switch with the subscript g or according to the parameters G_{1H} and T_{1H}, G_{2H} and T_{2H}, to G_{gH} and T_{gH} in the setup from the hot noise source to the waveguide switch. In the block diagram of figure 4, only two two-port networks are shown in each side of the setup between the noise source and the switch. The number of two-port networks depends, however, on the number of reflection locations and the number of elements having different loss characteristics and temperatures. To the right of the switch, the two-port networks are like those described in figure 1. Two noise-power indications P_C and P_H are possible, depending on the noise source to which the rest of the system is switched. By applying equation (2) to the two different noise-source possibilities, the following equation pair is obtained:

$$\left.\begin{aligned} P_H &= k\,\Delta f\left(T_H \prod_{n=1H}^{m} G_n + T_{1H} \prod_{n=2H}^{m} G_n + T_{2H} \prod_{n=3H}^{m} G_n + \ldots + T_k \prod_{n=k+1}^{m} G_n + \ldots + T_m\right) \\ P_C &= k\,\Delta f\left(T_C \prod_{n=1C}^{m} G_n + T_{1C} \prod_{n=2C}^{m} G_n + T_{2C} \prod_{n=3C}^{m} G_n + \ldots + T_k \prod_{n=k+1}^{m} G_n + \ldots + T_m\right) \end{aligned}\right\} \tag{15}$$

The second subscripts H and C refer to the networks connected to the hot noise source and cold noise source, respectively, as also indicated in figure 4. The portion of the output noise power at the end of the cascaded networks due to any one of the two-port networks following the switch with the subscript g can be calculated by inserting equations (15) into equation (13) and solving for the selected network noise power. The symbol T_k denotes the equivalent temperature of the output noise power caused by the noise generated in the kth network. This network is assumed to be the amplifier under test. The portion of the indicated noise power due to this selected amplifier is then defined by the following output noise temperature:

$$T_k \prod_{n=k+1}^{m} G_n = \frac{T_H \prod\limits_{n=1H}^{m} G_n + T_{1H} \prod\limits_{n=2H}^{m} G_n + \ldots + T_{k-1} \prod\limits_{n=k}^{m} G_n + T_{k+1} \prod\limits_{n=k+2}^{m} G_n + \ldots + T_m}{Y - 1}$$

$$- \frac{Y\left(T_C \prod\limits_{n=1C}^{m} G_n + T_{1C} \prod\limits_{n=2C}^{m} G_n + \ldots + T_{k-1} \prod\limits_{n=k}^{m} G_n + T_{k+1} \prod\limits_{n=k+2}^{m} G_n + \ldots + T_m\right)}{Y - 1} \tag{16}$$

In general, the noise characteristics of low-noise amplifiers are expressed in terms of input noise temperatures. Therefore, the portion of the indicated noise power due to the kth network has to be transformed in terms of noise temperature to the input of the amplifier under test by dividing both sides of equation (16) by $\prod\limits_{n=k}^{m} G_n$. The left side is then T_k/G_k and

$$T = \frac{T_k}{G_k} \tag{17}$$

where T is the input noise temperature of the amplifier under test. The right side of equation (16) likewise changes so that

$$T = \frac{T_H \prod\limits_{n=1H}^{k-1} G_n + T_{1H} \prod\limits_{n=2H}^{k-1} G_n + \ldots + T_{k-1} + T_{k+1} \frac{1}{G_k G_{k+1}} + \ldots + T_n \frac{1}{\prod\limits_{n=k}^{m} G_n}}{Y - 1}$$

$$- \frac{Y\left(T_C \prod\limits_{n=1C}^{k-1} G_n + T_{1C} \prod\limits_{n=2C}^{k-1} G_n + \ldots + T_{k-1} + T_{k+1} \frac{1}{G_k G_{k+1}} + \ldots + T_m \frac{1}{\prod\limits_{n=k}^{m} G_n}\right)}{Y - 1} \tag{18}$$

This equation shows that all noise temperatures due to noise generation along the cascaded networks are fictively transformed by the remaining portions of the gain series to the input of the amplifier under test. To simplify equation (18) for error-propagation calculations, these fictive quantities are introduced as modified noise temperatures. The sum of the terms with subscripts H to gH in equation (18) represents a modified noise temperature T_h due to noise generated in the hot noise source and the following networks up to the switch; that is,

$$T_h = T_H \prod_{n=1H}^{k-1} G_n + T_{1H} \prod_{n=2H}^{k-1} G_n + T_{2H} \prod_{n=3H}^{k-1} G_n + \dots + T_{gH} \prod_{n=g+1}^{k-1} G_n \tag{19}$$

The sum of the terms with subscripts C to gC in equation (18) represents a modified noise temperature T_c due to noise generated in the cold noise source and the following networks up to the switch; that is,

$$T_c = T_C \prod_{n=1C}^{k-1} G_n + T_{1C} \prod_{n=2C}^{k-1} G_n + T_{2C} \prod_{n=3C}^{k-1} G_n + \dots + T_{gC} \prod_{n=g+1}^{k-1} G_n \tag{20}$$

The noise generated between the switch network g and the amplifier under test (network k) has a modified noise temperature

$$T_x = T_{g+1} \prod_{n=g+2}^{k-1} G_n + \dots + T_{k-2}\, G_{k-1} + T_{k-1} \tag{21}$$

The modified temperatures $T_h + T_x$ and $T_c + T_x$ are the noise temperatures of the noise powers actually available to the amplifier under test. The remaining terms of equation (18) may be replaced by the modified temperature T_z as follows:

$$T_z = T_{k+1} \frac{1}{G_k G_{k+1}} + \dots + T_m \frac{1}{\prod_{n=k}^{m} G_n} \tag{22}$$

With this notation, equation (18) simplifies to

$$T = \frac{T_h + T_x + T_z - Y\left(T_c + T_x + T_z\right)}{Y - 1} \tag{23}$$

or

$$T = \frac{T_h - Y\,T_c}{Y - 1} - T_x - T_z \tag{24}$$

The frequently used expression for the effective noise temperature of a receiver system is

$$T_e = T + T_z \tag{25}$$

If the changes of the noise temperatures from T_H to T_h and from T_C to T_c are negligible and if T_x is small with respect to T_e, equation (24) changes, with substitution of equation (25), to the following well-known expression for the determination of the effective noise temperature from the measured Y-factor of the system under test:

$$T_e \approx \frac{T_H - Y\,T_C}{Y - 1} \tag{26}$$

The effective noise temperature T_e is an appropriate and convenient quantity for the evaluation of the performance of complete receiving systems. In many cases, however, such as laboratory investigations on low-noise preamplifiers, determination of the noise temperature of individual devices, and calibration of noise sources, it is important to determine the input noise temperature alone. Then, a careful determination of all the different effects shown in equation (18) must be made. How accurate this determination has to be can be evaluated with the calculation of error propagation.

ERROR ANALYSIS OF THE Y-FACTOR METHOD

The accuracy of the calculation of the input noise temperature by using the Y-factor measurement can be determined by means of the general linear error-propagation equation in the differential form

$$\xi = \frac{\dfrac{\partial f(x,y,z,\ldots)}{\partial x}\,\Delta x}{f(x,y,z,\ldots)} + \frac{\dfrac{\partial f(x,y,z,\ldots)}{\partial y}\,\Delta y}{f(x,y,z,\ldots)} + \frac{\dfrac{\partial f(x,y,z,\ldots)}{\partial z}\,\Delta z}{f(x,y,z,\ldots)} \ldots \tag{27}$$

or in the relative-error-notation form

$$\xi = \frac{f\left(\dfrac{x}{1 - \xi_x}, \dfrac{y}{1 - \xi_y}, \dfrac{z}{1 - \xi_z}, \ldots\right)}{f(x,y,z,\ldots)} - 1 \tag{28}$$

where ξ is the relative error of the function $f(x,y,z,...)$; Δx, Δy, and Δz represent the absolute errors; and ξ_x, ξ_y, and ξ_z are the relative errors of the variables x, y, and z. Some measuring errors enter into equations (7), (8), (11), and (12). From there, the errors propagate through equation (18) to equation (24). Because of the complexity of this error propagation, a step-by-step analysis from the measurement errors to the relative error of the input noise temperature will be made.

The first step in the error analysis is the determination of the relative error a_l of the mismatch factor α_l of the l th network from the measured reflection coefficients. Equation (12), which is a function of four independent variables (the absolute values of the reflection coefficient Γ and the respective phase angles b_{l1} and b_{l2}), is inserted into equation (28). The lengthy but straight-forward calculation is given in appendix A. For the most likely case that the relative amplitude errors γ_{l1} and γ_{l2} of $|\Gamma_{l1}|$ and $|\Gamma_{l2}|$ are equal and the relative phase errors β_{l1} and β_{l2} of the phase angles b_{l1} and b_{l2} are equal, the relative error of the mismatch factor of the l th network is

$$a_l = \left[1 - 2\gamma_{l1}\left(\frac{|\Gamma_{l1}|^2}{1 - |\Gamma_{l1}|^2} + \frac{|\Gamma_{l2}|^2}{1 - |\Gamma_{l2}|^2}\right)\right]$$

$$\left\{\frac{|\Gamma_{l1}|^2 \, |\Gamma_{l2}|^2 + 1 - 2|\Gamma_{l1}|\,|\Gamma_{l2}| \cos\left(b_{l1} + b_{l2}\right)}{1 + |\Gamma_{l1}|^2 \, |\Gamma_{l2}|^2 \left(1 + 4\gamma_{l1}\right) - 2|\Gamma_{l1}|\,|\Gamma_{l2}|\left(1 + 2\gamma_{l1}\right) \cos\left[\left(b_{l1} + b_{l2}\right)\left(1 + \beta_{l1}\right)\right]}\right\} - 1 \tag{29}$$

The second step is the calculation of the relative error g_l of the gain factor G of the example network l. Equation (7) is introduced into equation (27). Taking the derivative and rearranging yields

$$g_l = a_l - \lambda_l \tag{30}$$

where λ_l is the relative error of the dissipation loss L_l of the l th network. Because the relative error can be positive or negative, the maximum error is assumed to be

$$g_l = |a_l| + |\lambda_l| \tag{31}$$

The third step is the calculation of the relative error τ_l of the noise temperature T_l defined by equation (8). The relative error of the ambient temperature t_l is ϑ_l. Inserting equation (8) into equation (28) and rearranging yields

$$\tau_l = \left|\frac{\lambda_l}{L_l - 1}\right| + |\vartheta_l| \tag{32}$$

It is very important to measure small losses L_l with the highest possible accuracy to avoid large relative errors τ_l.

The fourth step is the determination of the relative errors of the modified noise temperatures T_h, T_c, T_x, and T_z defined by equations (19) to (22). The derivation of the relative error τ_h of the modified noise temperature T_h is presented in detail in appendix B with the aid of error-propagation equation (28). The result is

$$\tau_h = \frac{T_H \prod_{n=1H}^{k-1} G_n\left(\tau_H + \sum_{n=1H}^{k-1} g_n\right) + T_{1H} \prod_{n=2H}^{k-1} G_n\left(\tau_{1H} + \sum_{n=2H}^{k-1} g_n\right) + \ldots + T_{gH} \prod_{n=g+1}^{k-1} G_n\left(\tau_{gH} + \sum_{n=g+1}^{k-1} g_n\right)}{T_H \prod_{n=1H}^{k-1} G_n + T_{1H} \prod_{n=2H}^{k-1} G_n + \ldots + T_{gH} \prod_{n=g+1}^{k-1} G_n} \tag{33}$$

applying the same method of appendix B to the modified noise temperatures T_c, T_x, and T_z gives

$$\tau_c = \frac{T_C \prod_{n=1C}^{k-1} G_n\left(\tau_C + \sum_{n=1C}^{k-1} g_n\right) + T_{1C} \prod_{n=2C}^{k-1} G_n\left(\tau_{1C} + \sum_{n=2C}^{k-1} g_n\right) + \ldots + T_{gC} \prod_{n=g+1}^{k-1} G_n\left(\tau_{gC} + \sum_{n=g+1}^{k-1} g_n\right)}{T_C \prod_{n=1C}^{k-1} G_n + T_{1C} \prod_{n=2C}^{k-1} G_n + \ldots + T_{gC} \prod_{n=g+1}^{k-1} G_n} \tag{34}$$

$$\tau_x = \frac{T_{g+1} \prod_{n=g+2}^{k-1} G_n\left(\tau_{g+1} + \sum_{n=g+2}^{k-1} g_n\right) + \ldots + T_{k-2} G_{k-1}\left(\tau_{k-2} + g_{k-1}\right) + T_{k-1}\tau_{k-1}}{T_{g+1} \prod_{n=g+2}^{k-1} G_n + \ldots + T_{k-2} G_{k-1} + T_{k-1}} \tag{35}$$

$$\tau_z = \frac{\frac{T_{k+1}}{G_k G_{k+1}}\left(g_k + g_{k+1} + |\tau_{k+1}|\right) + \ldots + \frac{T_m}{\prod_{n=k}^{m} G_n}\left(\sum_{n=k}^{m} g_n + |\tau_m|\right)}{\frac{T_{k+1}}{G_k G_{k+1}} + \ldots + \frac{T_m}{\prod_{n=k}^{m} G_n}} \tag{36}$$

The fifth and final step in the error analysis is the determination of the relative error τ of the input noise temperature T of the device under test. Equation (23) is rearranged to yield

$$Y = \frac{T_h + T_x + T + T_z}{T_c + T_x + T + T_z} \tag{37}$$

Inserting this equation into equation (27) results in

$$\epsilon = \frac{\Delta T_h}{T_h + T_x + T + T_z} - \frac{\Delta T_c}{T_c + T_x + T + T_z}$$

$$+ \frac{T_c - T_h}{\left(T_h + T_x + T + T_z\right)\left(T_c + T_x + T + T_z\right)}\left(\Delta T_x + \Delta T + \Delta T_z\right) \tag{38}$$

where ϵ is the relative error of Y, and ΔT_c, ΔT_h, ΔT_x, ΔT_z, and ΔT are the absolute errors of the modified noise temperatures and of the input noise temperature of the device under test. Solving for ΔT in equation (38) and for T in equation (37) and then forming the ratio $\Delta T/T$ yields

$$\tau = \frac{\left(\frac{T_x}{T_c} + \frac{T_z}{T_c} + \frac{T}{T_c} + 1\right)\left(\frac{T_h}{T} + \frac{T_x}{T} + \frac{T_z}{T} + 1\right)}{\frac{T_h}{T_c} - 1}\,\epsilon + \frac{\frac{T_c}{T} + \frac{T_x}{T} + \frac{T_z}{T} + 1}{1 - \frac{T_c}{T_h}}\,\tau_h$$

$$+ \frac{\frac{T_h}{T} + \frac{T_x}{T} + \frac{T_z}{T} + 1}{\frac{T_h}{T_c} - 1}\,\tau_c + \frac{T_x}{T}\,\tau_x + \frac{T_z}{T}\,\tau_z \tag{39}$$

Only positive values are considered and therefore maximum errors are obtained. The first term in equation (39) has a minimum at

$$T = \sqrt{\left(T_h + T_x + T_z\right)\left(T_c + T_x + T_z\right)} \tag{40}$$

The other terms are hyperbolic functions in T; that is, the relative error increases rapidly with decreasing input noise temperatures. It can be seen from equations (39) and (40) that in order to move the minimum conditions of equation (40) to low input noise temperatures T, $T_x + T_z$ must be very small in comparison to T_h and T_c, the ratio T_h/T_c must be large, and the product $T_h T_c$ must be small. The denominator of the second term in equation (39) is not changed appreciably by the requirement that T_h/T_c be very large. Therefore, it is suggested to make $T > T_c$. The last three terms also become small under this condition. However, the most direct approach for minimizing τ is to make ϵ, τ_h, τ_c, τ_x, and τ_z small.

This error analysis of the input noise temperature of the device under test in the general Y-factor measuring setup of figure 4 showed not only that consideration of the errors of the mismatch factor, gain, and noise temperature is important but also that the ratios T_h/T_c and T/T_c have an effect on the minimization of error propagation.

APPLICATION TO NEGATIVE-RESISTANCE AMPLIFIERS

The general concept of Y-factor measurement was applied to the more specific case of a low-noise, negative-resistance amplifier, such as a maser, parametric amplifier, or tunnel-diode amplifier. A typical measuring setup is shown in figure 5 in the form of a block diagram. This diagram will be used as a guide to calculate the noise parameters in equations (7) to (22) for the numbered locations 1 to 12. The line elements, such as line switch, circulators, and amplifiers, are connected by means of rectangular waveguides or coaxial lines between the reference planes. For simplicity, the lossy sections of the circulators are also designated as waveguides. The reference planes 1 to 12 indicate locations of possible reflections.

The resistances of the noise sources, closely matched to the impedance of the waveguide, deliver to plane 1 the noise powers of the hot and cold noise sources. Because of reflections in plane 1 and losses in lines 1-2, the gain factors are

$$G_{1H} = \frac{\alpha_{1H}}{L_{1H}} \qquad G_{1C} = \frac{\alpha_{1C}}{L_{1C}} \tag{41}$$

Losses in lines 1-2 in the "hot" and "cold" sides of the setup generate noise powers which depend on the ambient temperatures t_{1H} and t_{1C}. A temperature gradient may exist in lines 1-2 so that the average temperatures

$$t_{1H} = \frac{T_H + t'_{1H}}{2} \qquad t_{1C} = \frac{T_C + t'_{1C}}{2} \tag{42}$$

may be used as approximations if the losses are small. The symbol T_C is the temperature of the cold noise source, T_H is the temperature of the hot noise source, t'_{1H} is the temperature of the hot waveguide 1-2 in reference plane 2, and t'_{1C} is the temperature of the cold waveguide 1-2 in the same reference plane. The noise temperatures, according to equation (8), are

$$T_{1H} = \frac{T_H + t'_{1H}}{2}\left(1 - \frac{1}{L_{1H}}\right)$$

$$T_{1C} = \frac{T_C + t'_{1C}}{2}\left(1 - \frac{1}{L_{1C}}\right) \tag{43}$$

Reflection point 2 and switch 2-3 cause the next change. The gain factors are

$$G_{2H} = \frac{\alpha_{2H}}{L_{2H}} \qquad\qquad G_{2C} = \frac{\alpha_{2C}}{L_{2C}} \tag{44}$$

where α_{2H} is the mismatch factor in reference plane 2 of the hot line, α_{2C} is the mismatch factor of the cold line in the same plane, and L_{2H} and L_{2C} are the respective dissipation losses. The noise temperatures due to dissipation losses are

$$T_{2H} = t_{2H}\left(1 - \frac{1}{L_{2H}}\right) \qquad\qquad T_{2C} = t_{2C}\left(1 - \frac{1}{L_{2C}}\right) \tag{45}$$

where t_{2H} and t_{2C} are the ambient temperatures of the two positions of the switch. A distinction between hot and cold lines is not necessary after reference plane 3. The respective noise temperature and gain factor for the next line are

$$T_3 = t_3\left(1 - \frac{1}{L_3}\right) \qquad\qquad G_3 = \frac{\alpha_3}{L_3} \tag{46}$$

Waveguide 4-5 is the circulator section between ports 1 and 2 with the noise temperature and gain factor

$$T_4 = t_4\left(1 - \frac{1}{L_4}\right) \qquad\qquad G_4 = \frac{\alpha_4}{L_4} \tag{47}$$

From port 2 to the input of the amplifier under test, the noise temperature and gain factor are

$$T_5 = t_5\left(1 - \frac{1}{L_5}\right) \qquad\qquad G_5 = \frac{\alpha_5}{L_5} \tag{48}$$

For negative-resistance amplifiers, such as masers and cooled parametric amplifiers immersed in liquid coolant, the temperature t_5 must be approximated as shown in equations (42) or may be more accurately calculated as reported in reference 7. The amplifier under test has the gain G_6. Introducing equations (41) to (48) into equations (19) and (20) yields the following modified noise temperatures T_h and T_c for the setup under consideration:

$$T_h = T_H \frac{\alpha_{1H}\alpha_{2H}\alpha_3\alpha_4\alpha_5}{L_{1H}L_{2H}L_3L_4L_5} + \frac{T_H + t'_{1H}}{2}\left(1 - \frac{1}{L_{1H}}\right)\frac{\alpha_{2H}\alpha_3\alpha_4\alpha_5}{L_{2H}L_3L_4L_5} + t_{2H}\left(1 - \frac{1}{L_{2H}}\right)\frac{\alpha_3\alpha_4\alpha_5}{L_3L_4L_5} \tag{49}$$

$$T_c = T_C \frac{\alpha_{1C}\alpha_{2C}\alpha_3\alpha_4\alpha_5}{L_{1C}L_{2C}L_3L_4L_5} + \frac{T_C + t'_{1C}}{2}\left(1 - \frac{1}{L_{1C}}\right)\frac{\alpha_{2C}\alpha_3\alpha_4\alpha_5}{L_{2C}L_3L_4L_5} + t_{2C}\left(1 - \frac{1}{L_{2C}}\right)\frac{\alpha_3\alpha_4\alpha_5}{L_3L_4L_5} \tag{50}$$

Applying equations (46) to (48) to equation (21) results in

$$T_x = t_3\left(1 - \frac{1}{L_3}\right)\frac{\alpha_4\alpha_5}{L_4L_5} + t_4\left(1 - \frac{1}{L_4}\right)\frac{\alpha_5}{L_5} + t_5\left(1 - \frac{1}{L_5}\right) + T_{C2} \tag{51}$$

where T_{C2} is the equivalent noise temperature caused by the matched load at circulator 2.

In many measuring systems, an additional circulator or isolator is provided to stabilize reflection-type amplifiers. Circulator 2 of figure 5 is used for this purpose. Its third port is terminated with a matched impedance. This impedance generates the noise power

$$N_{C2} = k\,\Delta f\,t_{C2} \tag{52}$$

where t_{C2} is the temperature of the impedance. This noise power is delivered toward the noise sources. Portions of it are reflected at planes 4, 3, 2, and 1 and cause an additional noise power that appears in the input of the amplifier under test as

$$N'_{C2} = N_{C2}\left[\frac{(1-\alpha_4)\alpha_5}{L_4L_5} + \frac{(1-\alpha_3)\alpha_4\alpha_5}{L_3^2L_4L_5} + \frac{(1-\alpha_2)\alpha_3\alpha_4\alpha_5}{L_2^2L_3^2L_4L_5} + \frac{(1-\alpha_1)\alpha_2\alpha_3\alpha_4\alpha_5}{L_1^2L_2^2L_3^2L_4L_5}\right] \tag{53}$$

The first term in the brackets represents the portion reflected at plane 4 and attenuated by the losses in line 4-5, by the reflection at plane 5, and by the losses in line 5-6. The second term is the portion of the noise power reflected at plane 3 and attenuated by the line elements 3-4, 4-5, and 5-6 and by reflections at planes 4 and 5. The third term gives the portion reflected at plane 2 and attenuated according to the following ohmic and reflection losses. The last term represents the portion of the noise power reflected at plane 1 and attenuated by the dissipation and reflection losses up to reference plane 6. Equation (53) does not take into consideration multiple reflections at planes 5, 4, 3, 2, and 1. This omission is permissible because the reflections in those planes are very small in measuring setups for low-noise amplifiers. The ohmic losses and the mismatch factors of equation (53) can also be neglected in most cases because of their slight influence on the small amount of reflected noise power. How small this influence is can be determined from the loss and gain data of table I. Then equation (53) simplifies to

$$N'_{C2} = N_{C2}\left(4 - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_4\right) \tag{54}$$

Another portion of N_{C2} leaks through circulator 1 against the direction of power transfer, according to the isolation $\frac{1}{g_{C1}}$, toward the amplifier under test and is given by

$$N''_{C2} = N_{C2} \frac{1}{g_{C1}} \frac{\alpha_5}{L_5} \tag{55}$$

Because this effect is not related to any power dissipation, no excess noise generation occurs. The reflection at plane 5 and the loss in line 5-6 can be neglected here because of the greater influence of $\frac{1}{g_{C1}}$. The sum of the noise powers of equations (54) and (55) has an equivalent noise temperature

$$T_{C2} = t_{C2}\left(4 - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_4 + \frac{1}{g_{C1}}\right) \tag{56}$$

The effects of the networks following the amplifier under test are considered in the modified temperature T_z. This is the first time that gain of the amplifiers of the measuring system enters into the calculation. Because the mismatch factors α are usually very close to unity for systems used to measure low input noise temperatures, they are neglected in comparison to the gain of the amplifier under test. Therefore, only the dissipation losses in lines 7-8, 8-9, and 9-10 are considered. The respective noise temperatures are

$$\left.\begin{aligned} T_7 &= t_5\left(1 - \frac{1}{L_7}\right) \\ T_8 &= t_8\left(1 - \frac{1}{L_8}\right) \\ T_9 &= t_9\left(1 - \frac{1}{L_9}\right) \end{aligned}\right\} \tag{57}$$

and the gain factors become

$$G_7 = \frac{1}{L_7} \qquad G_8 = \frac{1}{L_8} \qquad G_9 = \frac{1}{L_9} \tag{58}$$

where L_7 is the dissipation loss in the reverse direction of line 5-6. The preamplifier of figure 5 is another reflection-type amplifier. Circulator 3 separates the incident power from the reflected power. The noise temperature of the preamplifier is, according to equation (11),

$$T_{10} = \left(F_{10} - 1\right) t_o G_{10} \tag{59}$$

where t_o is the room temperature (292° K) and F_{10} is the noise figure. The increased gain provided by the preamplifier is required for keeping the noise contribution of the following networks small. As can be seen from equation (62), all noise terms of the networks following the preamplifier are divided by the combined gain $G_6 G_{10}$ of the amplifier under test and the preamplifier. The circulator section 11-12 contributes

$$T_{11} = t_{11}\left(1 - \frac{1}{L_{11}}\right) \qquad G_{11} = \frac{1}{L_{11}} \tag{60}$$

In most cases, a mixer-amplifier combination is necessary to convert the high frequency to a convenient 30-MHz or 60-MHz IF, as shown in figure 5. The mixer stage has the noise temperature

$$T_{12} = \left(F_{12} - 1\right) t_o G_{12} \tag{61}$$

The rest of the noise powers generated by the precision attenuator and the 30-MHz receiver are of no importance because the modified noise temperature T_z is not appreciably affected by this portion of the circuit after division by all gain factors according to equation (22). Substituting equations (57) to (61) into equation (22) results in

$$\begin{aligned} T_z = {} & t_5\left(1 - \frac{1}{L_7}\right)\frac{L_7}{G_6} + t_8\left(1 - \frac{1}{L_8}\right)\frac{L_7 L_8}{G_6} + t_9\left(1 - \frac{1}{L_9}\right)\frac{L_7 L_8 L_9}{G_6} \\ & + \left(F_{10} - 1\right) t_o \frac{L_7 L_8 L_9}{G_6} + t_{11}\left(1 - \frac{1}{L_{11}}\right)\frac{L_7 L_8 L_9 L_{11}}{G_6 G_{10}} + \left(F_{12} - 1\right) t_o \frac{L_7 L_8 L_9 L_{11}}{G_6 G_{10}} \end{aligned} \tag{62}$$

In general, the temperature t_8, t_9, and t_{11} are room temperatures and can be replaced by t_o. Equation (62) indicates that with high gain G_6, the first four terms become small. The effect of the last two terms is very small if the product of G_6 and G_{10} is sufficiently large. Also, the influence of the last three terms can be held small for low gain G_6 if the noise figure F_{10} is small and the gain G_{10} is sufficiently high.

The precision attenuator, together with the IF receiver and indicator, enables measurements of the Y-factor. Usually, the attenuator is calibrated in decibels, and the resetting of the indicator position by the attenuator variation after the noise sources are switched gives

$$10 \log Y = 10 \log P_H - 10 \log P_C = 10 \log \frac{P_H}{P_C} \tag{63}$$

The antilog of the reading is the Y-factor of equation (24).

If a measuring setup differs from that shown in figure 5, the number of terms shown in equations (49), (50), (51), and (62) must be modified according to the complexity of the system.

NUMERICAL EXAMPLES

The measuring system shown in figure 5 was used for the determination of the input noise temperature of parametric amplifiers cooled with liquid nitrogen. The two noise sources were maintained at $T_H = 390^\circ$ K and $T_C = 77^\circ$ K. All the reflection and ohmic losses of the lines, switch, and circulators were measured and are given in table I. The interconnections were coaxial rigid lines or cables. The separations of the losses into reflection losses and ohmic losses were achieved by the measuring method described in reference 4, pages 462 to 476. These numerical separations showed that the ohmic losses produced the major portion of the change from the noise-source temperature to the available noise temperature at the input of the amplifier under test. Line 5-6 was partially at the temperature t_5' of 77° K because of immersion into liquid nitrogen. Inserting the appropriate values of table I into equations (41) to (62) gives the modified noise temperatures which appear in the last column of table I. By substituting these values into equation (23), the following expression for the input noise temperature is obtained:

$$T = \frac{374.8 - Y\,130.4}{Y - 1} \tag{64}$$

It is obvious from equations (26) and (64) that the noise temperatures of the noise sources are modified toward the ambient temperature of the lossy elements. In this case, the ambient temperature was $t_o = 292^o$ K. If it is important to minimize these temperature changes, for example, to eliminate the errors in T_x, it is recommended that the switch of figure 5 be placed just in front of the amplifier under test. For this case, all lines between the noise sources and the amplifier under test, as well as circulator 1, must be doubled. Each circuit would then be kept at the temperature of the noise source to which it is connected. Another switch on the output ports of the two circulators replacing circulator 1 would be required to connect line 9-10.

In order to show how the accuracy of T is influenced by many factors, two cases, case A and case B, with different errors are postulated for three different temperature conditions. Case A refers to the measuring setup of figure 5 with the data of table I and with the following measurement errors:

$$\gamma_{l1} = \gamma_{l2} = 0.1$$

$$\beta_{l2} = \beta_{l2} = 0.06$$

$$10 \log(\lambda_n + 1) = 0.02 \text{ dB}$$

$$10 \log(g_6 + 1) = 10 \log(g_{10} + 1) = 0.1 \text{ dB}$$

$$\Delta T_H = \Delta T_C = 1^o \text{ K}$$

$$\Delta t_n = 2^o \text{ K}$$

$$\tau_{10} = 0.66 \times 10^{-2}$$

$$\tau_{12} = 2.3 \times 10^{-2}$$

$$\epsilon = 0.71 \times 10^{-2}$$

Case B also refers to figure 5 and the data of table I, but with the following lower errors:

$$\gamma_{l1} = \gamma_{l2} = 0.05$$

$$\beta_{l1} = \beta_{l2} = 0.03$$

$$10 \log(\lambda_n + 1) = 0.005 \text{ dB}$$

$$10 \log(g_6 + 1) = 10 \log(g_{10} + 1) = 0.01 \text{ dB}$$

$$\Delta T_C = 0.1^\circ \text{ K}$$

$$\Delta T_H = 1^\circ \text{ K}$$

$$\Delta t_n = 2^\circ \text{ K}$$

$$\tau_{10} = 0.66 \times 10^{-2}$$

$$\tau_{12} = 2.3 \times 10^{-2}$$

$$\epsilon = 0.71 \times 10^{-2}$$

The measurement accuracies of case A are not too high and are achievable with good instruments. The errors γ of the very low reflection coefficients of table I could be kept smaller than 10 percent with the method described in reference 4. The errors β of the phase angles could likewise be kept smaller than 6 percent. Equation (29), however, indicates that the smaller the reflection coefficient, the higher the error can be for the same error of the mismatch factor. A loss error λ of less than 0.02 dB is not difficult to realize. A loss error of this magnitude introduces large errors in the noise temperatures if the losses are very small and λ stays the same as equation (32) shows. The absolute temperature error of 1° K of the noise sources is no problem if boiling liquids or regulated heat sources are used. The absolute temperature error Δt_n is related to the variation of the room temperature and is approximately equal to the sensitivity of a good air-conditioning system. The symbols τ_{10} and τ_{12} are the relative errors of the noise temperatures of the preamplifier and the mixer derived from the 0.02-dB noise-figure accuracy of the preamplifier and the 0.1-dB noise-figure accuracy of the mixer. The error ϵ of the Y-factor is determined from the short-time gain stability of the system of 0.02 dB and from the accuracy of the precision attenuator of 0.011 dB. The error assumptions in case A are reasonable for normal laboratory conditions. For case B, the requirements on the accuracies of the different quantities are more stringent but can be satisfied with precision instruments and very careful consideration of temperature conditions. The Y-factor error ϵ is kept conservative at the same value. It could be improved to 0.25×10^{-2} by using very stable amplifiers, a stable receiver, and a precision attenuator with light indicators. The following table shows the three noise-source conditions for both cases A and B:

Condition	T_C, °K	T_H, °K
1	77	390
2	4.2	77
3	4.2	390

The determination of the relative error τ of the input noise temperature is similar to the evaluation of equation (18) for the negative-resistance-amplifier condition of figure 5. The same index restrictions according to figure 5 are applied to error-propagation equations (29) to (36). After insertion into these equations of the data of table I and the error data for cases A and B (conditions 1 to 3), the data of table II are obtained. The introduction of the data of table II into equation (39) results in the curves of figure 6, which represent the maximum input-noise-temperature error in percent as a function of the input noise temperature. Curve 1 of case B is for the condition which was realized in the measuring setup of figure 5 for parametric amplifiers immersed in liquid nitrogen. The other curves are for simulated conditions to show the different influences of the hot- and cold-noise-source temperatures and the errors. The curves illustrate very well that the ratio T_H/T_C is not as important as it seemed to be at first glance at equation (39). For curve 2 the ratio T_H/T_C is about 18, and for curve 1 it is about 5. It is obvious in both cases A and B that the use of liquid helium as the coolant for the cold noise source in condition 3 lessens the error considerably for $T < 100^\circ$ K but only slightly for $T > 100^\circ$ K. A comparison of cases A and B shows very well that the improvement in measuring accuracies of the basic quantities results in lower errors in the region of input noise temperatures above 50° K but does not contribute much in the region below 20° K. The very rapid increase in errors around 30° K is alarming. This characteristic of error propagation should not be underestimated, especially in maser investigations. The relative errors would become smaller if the test setup of figure 5 were instrumented for frequencies of 2 to 10 GHz with low-loss rectangular waveguides and improved circulators. Above this frequency, the relative noise-temperature error increases again because of larger waveguide losses and increased measuring errors.

These examples demonstrated that stable and accurate test instruments, stable noise sources, and the choice of one noise source of low temperature result in a measuring system that is fairly accurate in spite of all the error possibilities. To utilize this accuracy of the Y-factor method, it is important to consider carefully all effects previously mentioned, as is demonstrated in the following example. When all the reflections, ohmic losses, temperature uncertainties, and so forth, in equations (41) to (62) are neglected, as is frequently done for the measurement of relatively high input noise temperatures (greater than 1000° K) and sometimes also for low input noise temperatures, the input noise temperature of equation (18) reduces to a false input noise temperature as follows:

$$T_f = \frac{T_H - YT_C}{Y - 1} - \frac{F_{10} - 1}{G_6} - \frac{F_{12} - 1}{G_6 G_{10}} \tag{65}$$

Using in equation (65) the values in table I for F_{10}, F_{12}, G_6, and G_{10} and the unchanged noise-source temperatures T_H and T_C of condition 1 results in

$$T_f = \frac{390 - Y77}{Y - 1} - 8 \tag{66}$$

The relative error is expressed by the equation

$$p = \frac{T_f - T}{T} 100 \tag{67}$$

By eliminating the output power ratio Y from equations (64) and (66) and satisfying equation (67), the relative error p for noise-temperature condition 1 is obtained. The corresponding curve is shown as a dashed line in figure 6 and is identified by equation (67). In both cases A and B, the differences between the curve of the relative error τ of condition 1 and the curve of the relative error p of the simplified equation (65) are moderate to the right of $T = 800^\circ$ K and increase rapidly as T approaches 50° K. For example, curve 1 of case B indicates a maximum error of only 11 percent at $T = 100^\circ$ K, for which the relative error p is about 110 percent. To the left of $T = 50^\circ$ K, all curves show considerable errors which are not worth being compared in this paper.

It must be emphasized that the errors expressed by equation (39) and presented in figure 6 are the maximum errors which result from the most unfavorable error propagation. That is, all errors are derived from the maximum uncertainties of the measurements and, in the process of following the error propagation through to equation (39), all relative errors are added up as absolute values. In a complex measuring system with many components, such as the system shown in figure 5, it is very likely that the calculated maximum error never occurs. However, the calculated error for the simplified equation (65) is fixed and does not depend on probability conditions.

The foregoing examples indicate again not only that all the effects before and after the amplifier under test have to be included in the determination of the input noise temperature from the Y-factor measurement but also that these effects must be determined with the highest possible accuracy, especially for amplifiers with noise temperatures less than 100° K.

CONCLUDING REMARKS

The present state of the art in building stable systems, precision attenuators, and stable noise sources results in an error of the Y-factor method of less than 10 percent for an input noise temperature higher than 100° K when noise-source temperatures of 77° K and 390° K are used. To achieve this accuracy, all the effects which influence the noise power must be included when the Y-factor method is used. The numerical examples demonstrated the value in considering all elements which affect the calibrated noise power along the transfer path from the noise sources to the amplifier under test. The best way to avoid a drastic change from the noise-source temperature to the available noise temperature at the input of the amplifier under test is to keep the losses as small as possible, especially the ohmic losses, which produce the major portion of the change. An error analysis of the numerical examples revealed that the maximum error of the input noise temperature can be kept small by decreasing the measuring errors of the basic quantities of the measuring system. However, the error increases rapidly when input noise temperatures less than 50° K are approached. A comparison between the input noise temperature determined by considering all influencing effects and the input noise temperature determined without considering these effects for a typical low-noise measuring system for reflection-type amplifiers revealed a considerable difference.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., September 13, 1968,
125-22-03-02-23.

APPENDIX A

ERROR PROPAGATION FROM MEASURED REFLECTION COEFFICIENT TO MISMATCH FACTOR

The determination of the reflection coefficient consists, in most cases, of the measurement of the absolute value and of the phase angle. For reasons of simplifying the notation of the following equations, the absolute values of the reflection coefficient will be Γ_{l1} and Γ_{l2}, in deviation from the notation in equation (12). The phase angles are b_{l1} and b_{l2}. The calculation of the error propagation from the measured reflection coefficient of the lth reflection location (fig. 3) to the mismatch factor α_l is performed with the aid of the error equation

$$\Gamma = \frac{\Gamma_{measured}}{1 - \gamma} \tag{A1}$$

In other words, the value of Γ to be considered in the calculation is uncertain by the factor $1 - \gamma$, where γ, the relative error of Γ, is

$$\gamma = \frac{\Delta \Gamma}{\Gamma} \tag{A2}$$

The same relative-error concept applies to the phase angle b_{l1} and b_{l2}. With the new notation, equation (12) changes to

$$\alpha_l = \frac{\left(1 - \Gamma_{l1}^2\right)\left(1 - \Gamma_{l2}^2\right)}{\left|1 - \Gamma_{l1}\,\Gamma_{l2}\, e^{j\left(b_{l1}+b_{l2}\right)}\right|^2} \tag{A3}$$

The quantity α_l', which contains the errors of the reflection-coefficient measurement, is

$$\alpha_l' = \frac{\left[1 - \left(\frac{\Gamma_{l1}}{1 - \gamma_{l1}}\right)^2\right]\left[1 - \left(\frac{\Gamma_{l2}}{1 - \gamma_{l2}}\right)^2\right]}{\left|1 - \frac{\Gamma_{l1}\,\Gamma_{l2}}{\left(1 - \gamma_{l1}\right)\left(1 - \gamma_{l2}\right)} e^{j\left(\frac{b_{l1}}{1-\beta_{l1}} + \frac{b_{l2}}{1-\beta_{l2}}\right)}\right|^2} \tag{A4}$$

where β_{l1} and β_{l2} are the respective relative phase-angle errors of b_{l1} and b_{l2}. The errors are assumed to be small enough that any products in γ or β, such as

γ_{l1}^2, γ_{l2}^2, and $\gamma_{l1}\gamma_{l2}$, can be neglected. Thus, equation (A4) simplifies to

$$\alpha_l' = \frac{\left(1 - \frac{\Gamma_{l1}^2}{1 - 2\gamma_{l1}}\right)\left(1 - \frac{\Gamma_{l2}^2}{1 - 2\gamma_{l2}}\right)}{\left|1 - \frac{\Gamma_{l1}\Gamma_{l2}}{1 - \gamma_{l1} - \gamma_{l2}} e^{j\left[\frac{b_{l1}(1-\beta_{l2})+b_{l2}(1-\beta_{l1})}{1-\beta_{l1}-\beta_{l2}}\right]}\right|^2} \tag{A5}$$

As long as γ and β are small, the approximation

$$\frac{1}{1 - X} \approx 1 + X \tag{A6}$$

(where X is an expression in γ or β) is applicable and equation (A5) changes to

$$\alpha_l' = \frac{\left[1 - \Gamma_{l1}^2\left(1 + 2\gamma_{l1}\right)\right]\left[1 - \Gamma_{l2}^2\left(1 + 2\gamma_{l2}\right)\right]}{\left|1 - \Gamma_{l1}\Gamma_{l2}\left(1 + \gamma_{l1} + \gamma_{l2}\right)e^{j\left(b_{l1}+b_{l2}+b_{l1}\beta_{l1}+b_{l2}\beta_{l2}\right)}\right|^2} \tag{A7}$$

By multiplying in the numerator, neglecting the product term in γ, and rearranging, equation (A7) is simplified to

$$\alpha_l' = \frac{\left(1 - \Gamma_{l1}^2\right)\left(1 - \Gamma_{l2}^2\right) - 2\gamma_{l1}\Gamma_{l1}^2\left(1 - \Gamma_{l2}^2\right) - 2\gamma_{l2}\Gamma_{l2}^2\left(1 - \Gamma_{l1}^2\right)}{\left|1 - \Gamma_{l1}\Gamma_{l2}\left(1 + \gamma_{l1} + \gamma_{l2}\right)e^{j\left(b_{l1}+b_{l2}+b_{l1}\beta_{l1}+b_{l2}\beta_{l2}\right)}\right|^2} \tag{A8}$$

The relative error of α_l is the ratio of the difference between the uncertain value α_l' and the true value α_l to the true value α_l and is expressed as

$$a_l = \frac{\Delta\alpha_l}{\alpha_l} = \frac{\alpha_l' - \alpha_l}{\alpha_l} = \frac{\alpha_l'}{\alpha_l} - 1 \tag{A9}$$

Introducing equations (A3) and (A8) into equation (A9) and rearranging results in

$$a_l = -1 + \left[1 - 2\gamma_{l1}\left(\frac{\Gamma_{l1}^2}{1 - \Gamma_{l1}^2}\right) - 2\gamma_{l2}\left(\frac{\Gamma_{l2}^2}{1 - \Gamma_{l2}^2}\right)\right]\left|\frac{1 - \Gamma_{l1}\Gamma_{l2}e^{j\left(b_{l1}+b_{l2}\right)}}{1 - \Gamma_{l1}\Gamma_{l2}\left(1 + \gamma_{l1} + \gamma_{l2}\right)e^{j\left(b_{l1}+b_{l2}+b_{l1}\beta_{l1}+b_{l2}\beta_{l2}\right)}}\right|^2 \tag{A10}$$

Squaring the absolute-value term in equation (A10) yields

$$a_l = -1 + \left[1 - 2\gamma_{l1}\left(\frac{\Gamma_{l1}^2}{1 - \Gamma_{l1}^2}\right) - 2\gamma_{l2}\left(\frac{\Gamma_{l2}^2}{1 - \Gamma_{l2}^2}\right)\right] \times$$
$$\left[\frac{\Gamma_{l1}^2 \Gamma_{l2}^2 + 1 - 2\Gamma_{l1}\Gamma_{l2}\cos\left(b_{l1} + b_{l2}\right)}{1 + \Gamma_{l1}^2 \Gamma_{l2}^2\left(1 + 2\gamma_{l1} + 2\gamma_{l2}\right) - 2\Gamma_{l1}\Gamma_{l2}\left(1 + \gamma_{l1} + \gamma_{l2}\right)\cos\left(b_{l1} + b_{l2} + b_{l1}\beta_{l1} + b_{l2}\beta_{l2}\right)}\right] \tag{A11}$$

This expression appears to be rather complex and should be evaluated for all reflection coefficients in question before being used in other equations.

APPENDIX B

ERROR PROPAGATION FROM GAIN FACTOR AND NOISE TEMPERATURE TO MODIFIED NOISE TEMPERATURE

The modified noise temperatures T_h, T_c, T_x, and T_z have possible uncertainties in the noise temperature and gain factor. The maximum deviation from the true value can be determined by introducing the uncertainty factors related to g_n and τ_n according to the following equations:

$$\left.\begin{aligned} G_n &= \frac{G_{n,\text{measured}}}{1 - g_n} \\ T_n &= \frac{T_{n,\text{measured}}}{1 - \tau_n} \end{aligned}\right\} \tag{B1}$$

That is, the value of G_n or T_n to be considered in the calculation is uncertain by the factor $1 - g_n$ or $1 - \tau_n$, where g_n, the relative error of G_n, and τ_n, the relative error of T_n, are

$$\left.\begin{aligned} g_n &= \frac{\Delta G_n}{G_n} \\ \tau_n &= \frac{\Delta T_n}{T_n} \end{aligned}\right\} \tag{B2}$$

Applying the notation of equation (B1) to the first term of equation (19) results in

$$\frac{T_H G_{1H} G_{2H} \cdot \cdot \cdot G_{k-1}}{\left(1 - \tau_H\right)\left(1 - g_{1H}\right)\left(1 - g_{2H}\right) \cdot \cdot \cdot \left(1 - g_{k-1}\right)} \tag{B3}$$

All relative-error product terms, such as $\tau_H g_{1H}$, $g_{1H} g_{2H}$, and $g_{1H}{}^2$, can be neglected if the errors are small (less than 10 percent). The following approximation can then also be applied:

$$\frac{1}{1 - \tau_H - g_{1H} - g_{2H} - \cdot \cdot \cdot} = 1 + \tau_H + g_{1H} + g_{2H} + \cdot \cdot \cdot \tag{B4}$$

The series in (B3) then changes to

$$T_H \prod_{n=1H}^{k-1} G_n \left(1 + \tau_H + \sum_{n=1H}^{k-1} g_n\right) \tag{B5}$$

Introducing this error-propagation concept of (B3) to (B5) into all terms of equation (19) results in the following maximum false value T_h' of the modified noise temperature T_h:

$$T_h' = T_H \prod_{n=1H}^{k-1} G_n \left(1 + \tau_H + \sum_{n=1H}^{k-1} g_n\right) + T_{1H} \prod_{n=2H}^{k-1} G_n \left(1 + \tau_{1H} + \sum_{n=2H}^{k-1} g_n\right) + \dots$$

$$+ T_{gH} \prod_{n=g+1}^{k-1} G_n \left(1 + \tau_{gH} + \sum_{n=g+1}^{k-1} g_n\right) \tag{B6}$$

The definition of the relative error of T_h is

$$\tau_h = \frac{\Delta T_h}{T_h} = \frac{T_h' - T_h}{T_h} = \frac{T_h'}{T_h} - 1 \tag{B7}$$

Inserting equations (19) and (B6) into equation (B7) yields

$$\tau_h = \frac{T_H \prod_{n=1H}^{k-1} G_n \left(\tau_H + \sum_{n=1H}^{k-1} g_n\right) + T_{1H} \prod_{n=2H}^{k-1} G_n \left(\tau_{1H} + \sum_{n=2H}^{k-1} g_n\right) + \dots + T_{gH} \prod_{n=g+1}^{k-1} G_n \left(\tau_{gH} + \sum_{n=g+1}^{k-1} g_n\right)}{T_H \prod_{n=1H}^{k-1} G_n + T_{1H} \prod_{n=2H}^{k-1} G_n + \dots + T_{gH} \prod_{n=g+1}^{k-1} G_n} \tag{B8}$$

A numerical evaluation of this complex equation is suggested for further calculation of the relative error τ of the input noise temperature T.

The same method presented in detail herein applies to the calculation of the relative errors τ_c, τ_x, and τ_z of the modified temperatures T_c, T_x, and T_z.

REFERENCES

1. Blackwell, Lawrence A.; and Kotzebue, Kenneth L.: Semiconductor-Diode Parametric Amplifiers. Prentice-Hall, Inc., 1961.

2. Merlo, D.; Houghton, E. W.; and Halford, G. J.: Effect of Some Component Tolerances and Measuring Errors on Noise Measurements. Electron. Lett., vol. 1, no. 9, Nov. 1965, pp. 250-251.

3. Van der Ziel, Aldert: Noise. Prentice-Hall, Inc., 1954.

4. Ginzton, Edward L.: Microwave Measurements. McGraw-Hill Book Co., Inc., 1957.

5. Otoshi, T. Y.: RF Techniques: Mismatch Errors on Noise Temperature Measurements. Supporting Research and Advanced Development. Space Programs Sum. No. 37-40, Vol. IV (Contract No. NAS7-100), Jet Propulsion Lab., California Inst. Technol., Aug. 31, 1966, pp. 190-197.

6. Miller, C. K. S.; Daywitt, W. C.; and Arthur, M. G.: Noise Standards, Measurements, and Receiver Noise Definitions. Proc. IEEE, vol. 55, no. 6, June 1967, pp. 865-877.

7. Wells, J. S.; Daywitt, W. C.; and Miller, C. K. S.: Measurement of Effective Temperatures of Microwave Noise Sources. IEEE Trans. Instrum. Meas., vol. IM-13, no. 1, Mar. 1964, pp. 17-28.

TABLE I.- LOSSES, GAINS, AND NOISE TEMPERATURES OF A MEASURING SYSTEM FOR DETERMINATION OF INPUT NOISE TEMPERATURE OF NEGATIVE-RESISTANCE AMPLIFIERS

Network	Reflection coefficient, Γ_{l1}	Reflection coefficient, Γ_{l2}	Mismatch factor, α_n	Loss, L_n	Gain factor, G_n	Ambient temperature, t_n, °K	Noise temperature, T_n, °K	T_n referred to input of device under test, °K	Modified noise temperature, °K
C						77	77	61.1	
1C	$0.02e^{j180^\circ}$	0	0.9996	1.0235	0.9757	184.5	4.2	3.4	
2C	0	0	1	1.0282	0.9726	292	8.0	6.7	T_c = 71.2
H						390	390	276.7	
1H	$0.01e^{j180^\circ}$	$0.02e^{j0^\circ}$	0.99857	1.0728	0.9308	341	23.1	17.6	
2H	0	0	1	1.0963	0.9122	292	25.6	21.3	T_h = 315.6
3	$0.02e^{j60^\circ}$	$0.06e^{-j160^\circ}$	0.99566	1.0261	0.9703	292	7.7	6.6	
4	$0.05e^{-j150^\circ}$	$0.01e^{-j60^\circ}$	0.99676	1.0917	0.9130	292	24.5	22.9	
5	$0.01e^{j100^\circ}$	$0.04e^{j10^\circ}$	0.99802	1.07	0.9328	184.5	12.1	12.1 T_{C2} = 5.3	T_x = 46.9
6					100	77			
7				1.07	0.9346	184.5	12.1	0.13	
8				1.0686	0.9357	292	18.7	0.21	
9				1.223	0.8177	292	53.3	0.71	
10					50	292	711 F_{10} = 5.38 db	9.95	
11				1.092	0.9322	292	24.5	0.007	
12						292	4310 F_{12} = 12 db	1.29	T_z = 12.3

TABLE II.- MODIFIED NOISE TEMPERATURES AND RELATIVE ERRORS

FOR CONDITIONS 1 TO 3 OF CASES A AND B

Case	Condition	T_c	τ_c	T_h	τ_h	T_x	τ_x	T_z	τ_z	ϵ
A	1	71.2	6.72×10^{-2}	315.6	3.56×10^{-2}	46.9	8.48×10^{-2}	12.3	14.95×10^{-2}	0.71×10^{-2}
	2	12.8	21.8	86.8	5.41	46.9	8.48	12.3	14.95	.71
	3	12.8	21.8	315.6	3.56	46.9	8.48	12.3	14.95	.71
B	1	71.2	1.68×10^{-2}	315.6	1.22×10^{-2}	46.9	2.57×10^{-2}	12.3	1.30×10^{-2}	0.71×10^{-2}
	2	12.8	4.9	86.8	1.47	46.9	2.57	12.3	1.30	.71
	3	12.8	4.9	315.6	1.22	46.9	2.57	12.3	1.30	.71

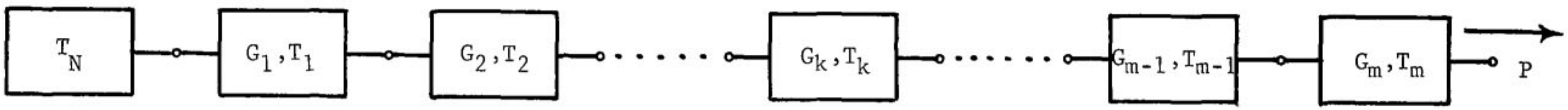

Figure 1.- Block diagram for determination of noise temperatures in cascaded networks.

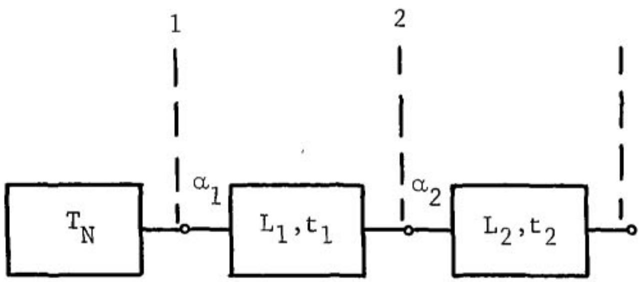

Figure 2.- Block diagram for networks with reflection and dissipation losses.

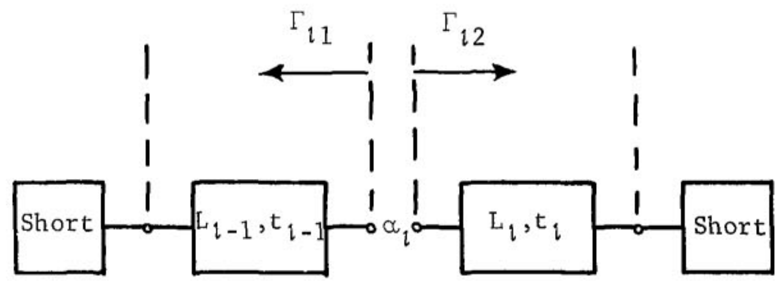

Figure 3.- Block diagram for derivation of mismatch factor.

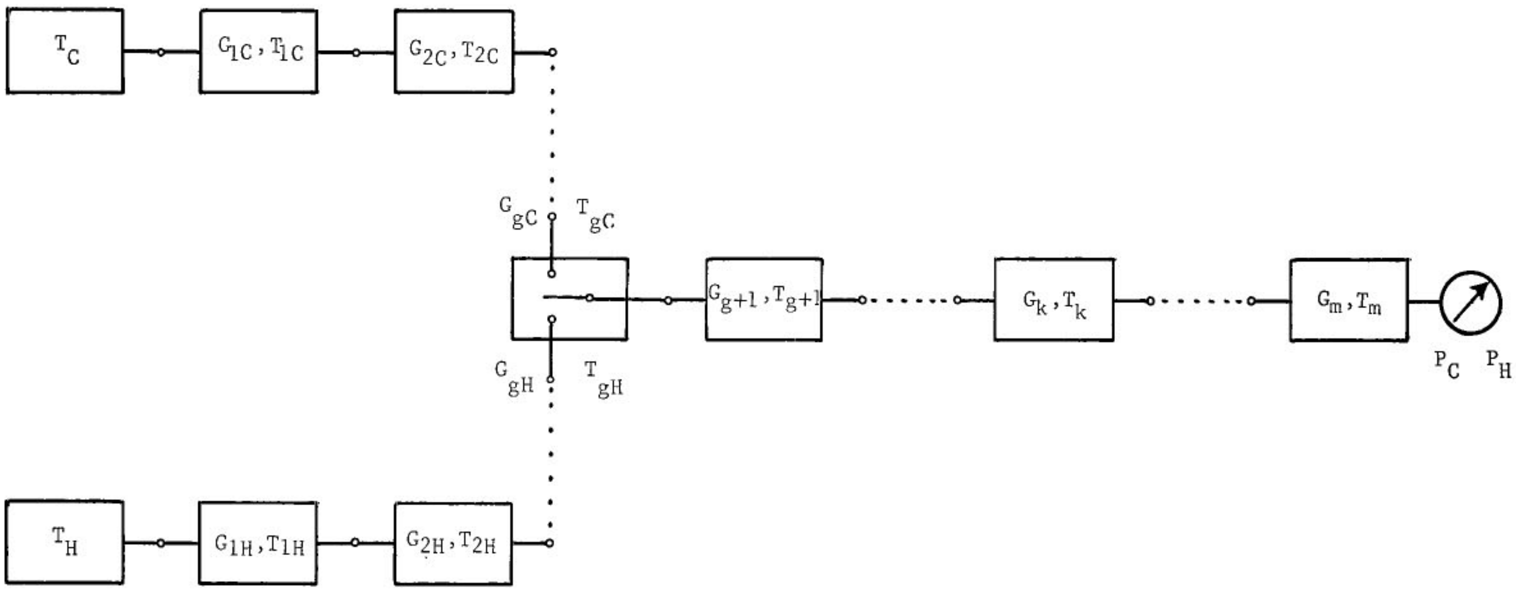

Figure 4.- Block diagram for Y-factor measurement.

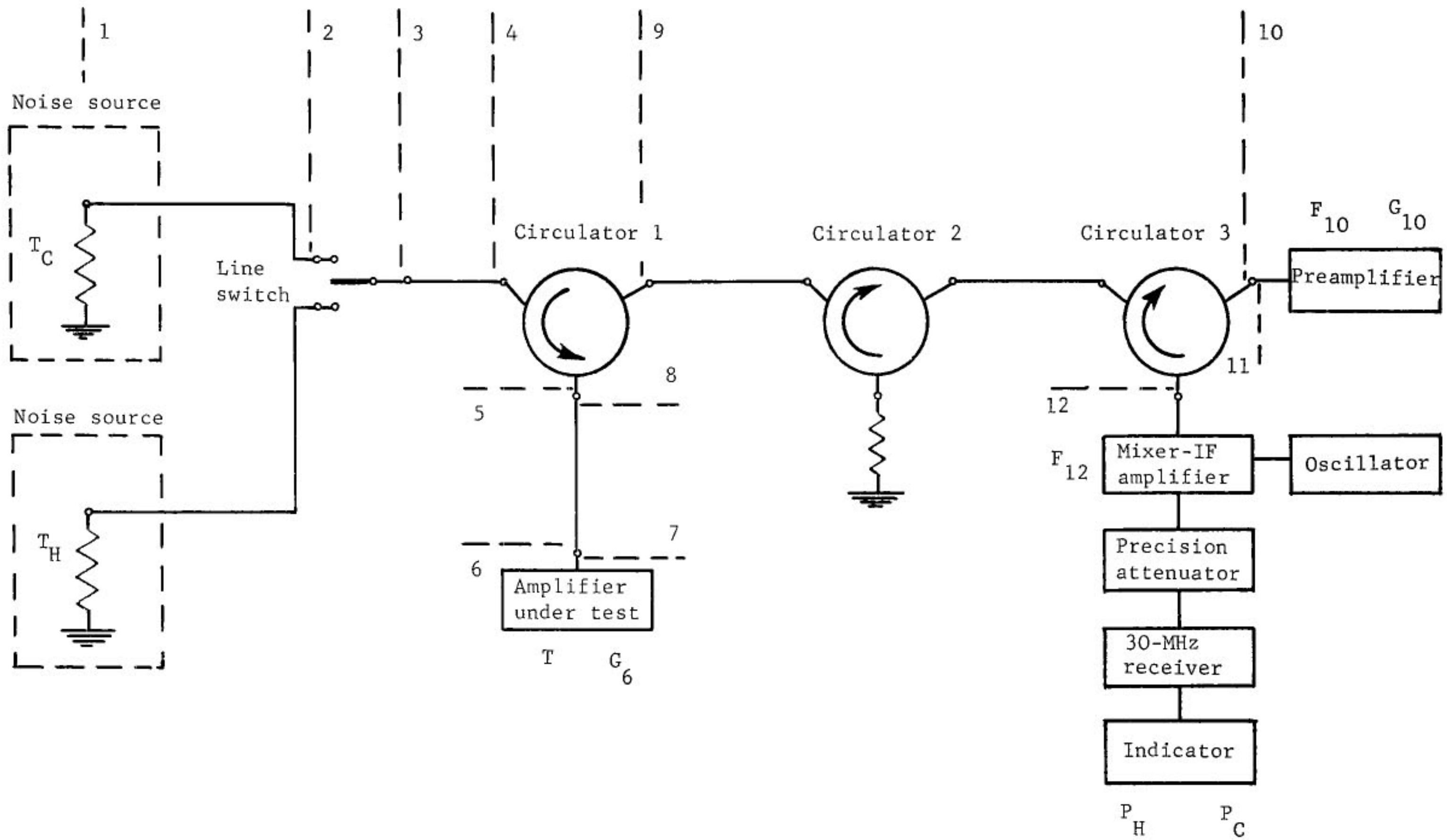

Figure 5.- Block diagram of input-noise measuring system for negative-resistance amplifier.

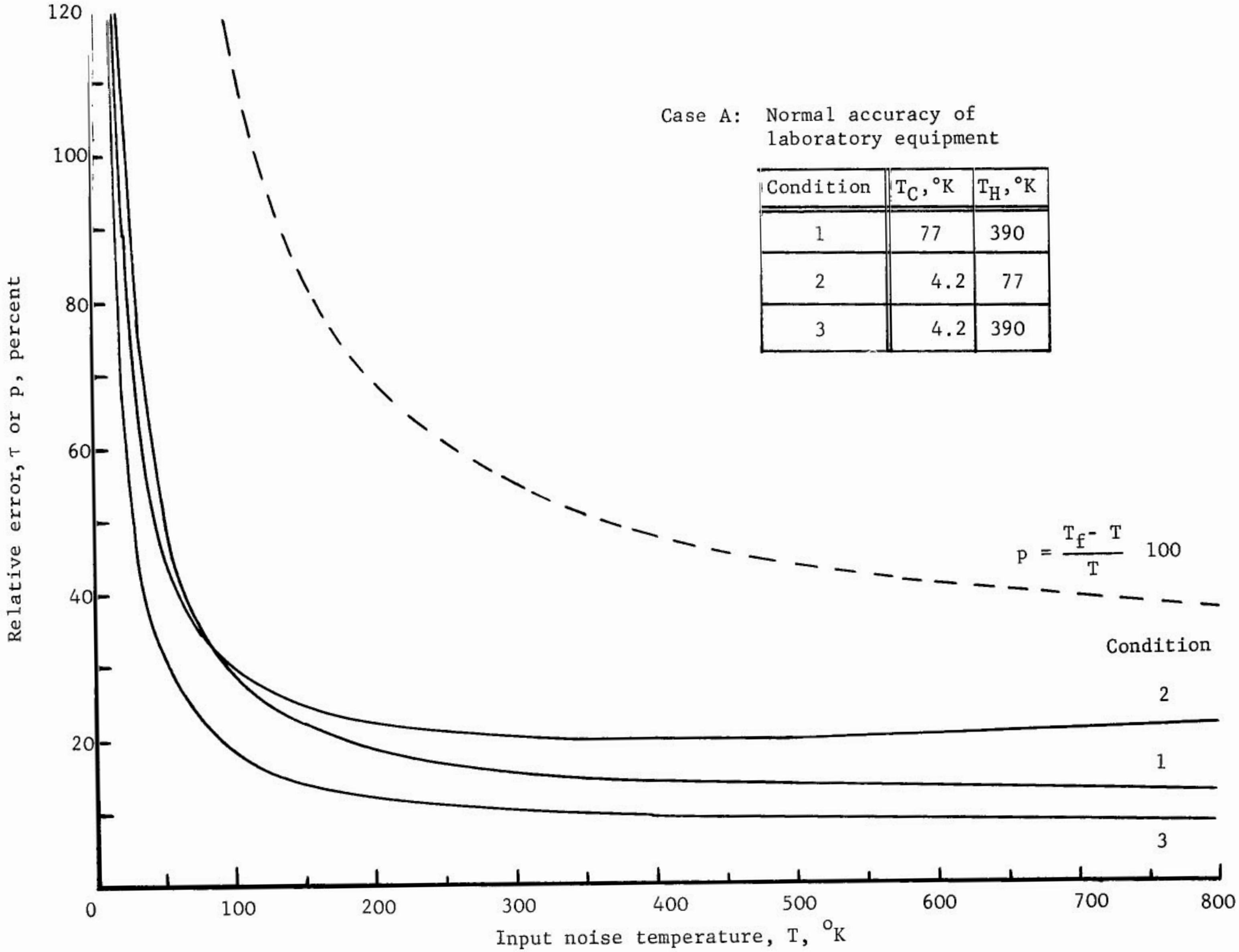

Figure 6.- Maximum input-noise-temperature error as a function of input noise temperature.

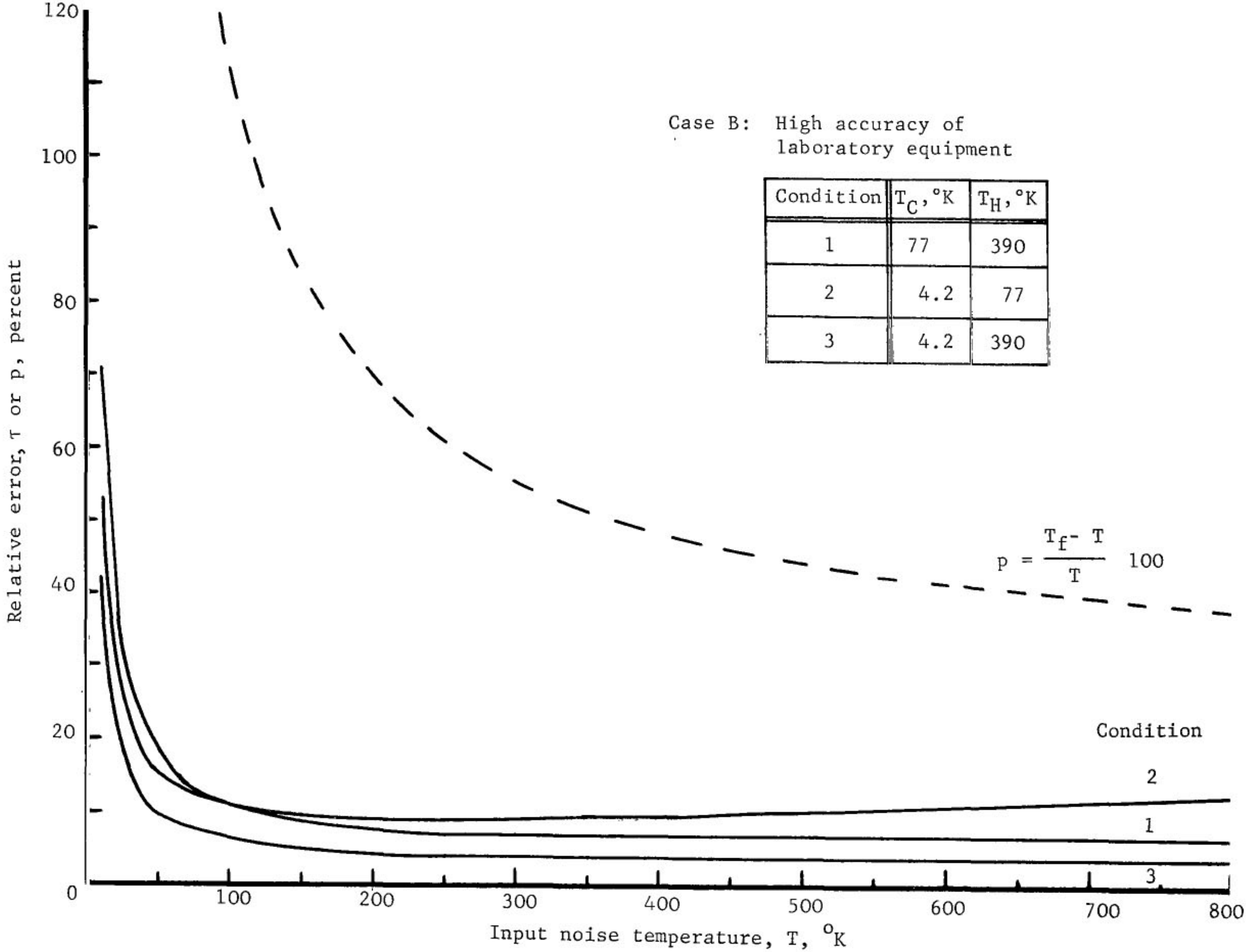

Figure 6.- Concluded.

NASA-Langley, 1969 — 7 L-5695

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Notes, and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546